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Some More Results on Intuitionistic Semi Open Sets

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ABSTRACT
The purpose of this paper is to define and study the intuitionistic semi open sets in intuitionistic topological space. Also some properties of this set are discussed.
Keywords - : Intuitionistic semi open set, Intuitionistic limit point, Intuitionistic interior, Intuitionistic closure, Intuitionistic topological space.

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [15], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov[3] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [7] introduced the notion of intuitionistic fuzzy topological spaces. The concept of intuitionistic set in topological space was first introduced by Coker [8]. He has studied some fundamental topological properties on intuitionistic sets. Later he studied intuitionistic connectedness [8] and intuitionistic points [9] in intuitionistic topological spaces. Open sets and open functions stand among the most important notion of the whole of mathematical science. Many different forms of open sets have been introduced over the years in general topology. Norman Levine [12] introduced semi-open sets and semi-continuity, O. Njastad [11] studied semi open sets, pre open sets and semi-preopen sets in general topological spaces. Shyamal Debnath [13] introduced the concept of intuitionistic fuzzy semi open and intuitionistic fuzzy semi continuous functions. In intuitionistic topology, intuitionistic open function was introduced by Coker [8] and later studied by Younis, Asmaa G.Raouf [14] and Duraisamy [10]. The fuzzy topological space was introduced by Chang [6] in 1968. Azad [4] has introduced the concept of fuzzy semi open and fuzzy semi closed sets in 1981.
In this paper we introduce and characterize the properties of intuitionistic semi open set, intuitionistic dense set and intuitionistic connected sets.

II. PRELIMINARIES

Throughout this paper X denote a non-empty set and (X,τ) represents the intuitionistic topological space (ITS). In this section, we shall present the fundamental definitions and propositions.

Definition 2.1. [6] An intuitionistic set (IS) A is an object having the form $<X, A_1, A_2>$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A, while A_2 is called the set of nonmembers of A. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of IS's in X, where $A_i = \left\langle X,\ A_i^{(1)}, A_i^{(2)} \right\rangle$ then

a) $\underset{\sim}{\varphi} = \langle X, \varphi, X \rangle$, $\underset{\sim}{X} = \langle X, X, \varphi \rangle$
b) $A \subseteq B$ *if* $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$
c) $\overline{A} = \langle X, A_2, A_1 \rangle$
d) $A\text{-}B = A \cap \overline{B}$
e) $\cap A_i = \left\langle X, \cap A_i^{(1)}, \cup A_i^{(2)} \right\rangle$ and $\cup A_i = \left\langle X, \cup A_i^{(1)}, \cap A_i^{(2)} \right\rangle$

Definition 2.2 . [6] An intuitionistic topological space (ITS) on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

i) (T_1) $\underset{\sim}{\varphi},\ \underset{\sim}{X} \in \tau$,
ii) (T_2) $G_1 \cap G_2 \in \tau$ for $G_1, G_2 \in \tau$,
iii) (T_3) $\cup G_i \in \tau$ for any arbitrary family $\{ G_i : i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called intuitionistic topological space and any intuitionistic set in τ is known as an intuitionistic open set (IOS) in X, and the complement of X is known as intuitionistic closed set (ISC) in X.

Definition 2.3. [6] Let (X,τ) be an intuitionistic topological space and $\langle X, A_1, A_2 \rangle$ be an intuitionistic set in X. Then the intuitionistic interior and intuitionistic closure of *A* are defined by

$$I\,\mathrm{int}(A) = \cup\{G / G \text{ is an IOS in } X \text{ and } G \subseteq A\}$$

$$Icl(A) = \cap\{K / K \text{ is an ICS in } X \text{ and } A \subseteq K\}$$

Definition 2.4. [7] Let X be a nonempty set and $p \in X$ a fixed element in *X*. Then the IS $\underset{\sim}{p}$ defined by $\underset{\sim}{p} = \langle X, \{p\}, \{p\}^c \rangle$ is called an intuitionistic point (IP) in *X*.

Definition 2.5. [1] Let (X,τ) be intuitionistic topological space and $\langle X, A_1, A_2 \rangle$ be an intuitionistic set in *X*. Then the set A is called intuitionistic dense in X if $Icl(A) = X$. Also a subset A of an ITS of *X* is said to be no where dense if the intuitionistic closure of *A* contains no intuitionistic interior points.

Definition 2.6. [1] An intuitionistic topological spaces (X,τ) is said to be intuitionistic submaximal spaces if every intuitionistic dense subset of *X* is an IOS in *X*.

Definition 2.7. [11] A set *A* in an intuitionistic topological space (X,τ) will be termed intuitionistic semi-open (ISO) if there exists an intuitionistic open set U such that $U \subseteq A \subseteq Icl(U)$. Also A is said to be an intuitionistic semi closed if its complement is intuitionistic semi-open.

Definition 2.8. [11] A set *A* in an intuitionistic topological space (X,τ) is said to be intuitionistic pre open (IPO) if $A \subseteq I\,\mathrm{int}(Icl(A))$ and intuitionistic pre closed set if $Icl(I\,\mathrm{int}(A)) \subseteq A$.

Definition 2.9. [8] A set *A* in an intuitionistic topological space (X,τ) is said to be intuitionistic α-open if $A \subseteq I\,\mathrm{int}(Icl(I\,\mathrm{int}(A)))$ and intuitionistic α- closed set if $Icl(I\,\mathrm{int}(Icl(A))) \subseteq A$.

Definition 2.10. [11] A set *A* in an intuitionistic topological space (X,τ) is said to be intuitionistic β-open (semi-pre open set) if $A \subseteq Icl(I\,\mathrm{int}(Icl(A)))$ and the intuitionistic β - closed set if $I\,\mathrm{int}(Icl(I\,\mathrm{int}(A))) \subseteq A$.

Definition 2.11.[8] *A*n intuitionistic topological space (X,τ) is said to be intuitionistic connected if $\underset{\sim}{X}$ cannot be expressed as the union of two nonempty disjoint intuitionistic open subsets of X. Otherwise (X,τ) is called intuitionistic disconnected.

Proposition . 2.12 [6] Let (X,τ) be an intuitionistic topological space and A, B be IS's in X. Then the following properties hold.

(1) $I\,\mathrm{int}(A) \subseteq A$ (2) $A \subseteq Icl(A)$
(3) $A \subseteq B \Longrightarrow I\,\mathrm{int}(A) \subseteq I\,\mathrm{int}(B)$
(4) $A \subseteq B \Longrightarrow Icl(A) \subseteq Icl(B)$
(5) $I\mathrm{int}(I\mathrm{int}(A)) = I\mathrm{int}(A)$
(6) $Icl(Icl(A)) = Icl(A)$
(7) $I\mathrm{int}(A \cap B) = I\mathrm{int}(A) \cap I\,\mathrm{int}(B)$
(8) $Icl(A \cup B) = Icl(A) \cup Icl(B)$
(9) $I\mathrm{int}(\underset{\sim}{X}) = \underset{\sim}{X}$, $I\mathrm{int}(\underset{\sim}{\phi}) = \underset{\sim}{\phi}$
(10) $Icl(\underset{\sim}{\phi}) = \underset{\sim}{\phi}$, $Icl(\underset{\sim}{X}) = \underset{\sim}{X}$

III. INTUITIONISTIC SEMI OPEN SETS

Proposition 3.1. Let *A* and A_α denote intuitionistic subsets of intuitionistic space *X*. Then

1) $\cup Icl(A_\alpha) \subseteq Icl(\cup A_\alpha)$
2) $\cup I\,\mathrm{int}(A_\alpha) \subseteq I\,\mathrm{int}(\cup A_\alpha)$

Theorem 3.2. A subset *A* in an intuitionistic topological space (X,τ) is an intuitionistic semi-open if and only if $A \subset Icl(I\,\mathrm{int}(A))$.

Proof . Necessity. Let A be intuitionistic semi-open. Then there exist an intuitionistic open set B, such that $B \subset A \subset Icl(B)$, but $B \subset I\,\mathrm{int}(A)$ and thus $Icl(B) \subset Icl(I\,\mathrm{int}(A))$. Hence $A \subset I\,cl(B) \subset Icl(I\,\mathrm{int}(A))$.

Sufficiency. Let $A \subset Icl(I\,\mathrm{int}(A))$. We have $I\,\mathrm{int}(A) \subset A$. Then for $I\,\mathrm{int}(A) \subset A \subset Icl(I\,\mathrm{int}(A))$. Taking *U=I* int(*A*), it becomes $U \subseteq A \subseteq Icl(U)$. This implies *A* is intutionistic semi open.

Theorem 3.3. Let $\{A_\alpha\}_{\alpha \in J}$ be a collection of intutionistic semi open in an ITS (X,τ). Then $\cup A_\alpha$ is intutionistic semi open.

Proof . Let $A_\alpha = \langle X, A_\alpha^1, A_\alpha^2 \rangle$, Since each A_α is intutionistic semi open, for each $\alpha \in J$ there exists an IS U_α such that $U_\alpha \subseteq A_\alpha \subseteq Icl(U_\alpha)$. Then

$$\cup_{\alpha \in J} U_\alpha \subseteq \cup_{\alpha \in J} A_\alpha \subseteq (\cup_{\alpha \in J} Icl(U_\alpha)) \subseteq Icl(\cup_{\alpha \in J}(U_\alpha)).$$

Remark 3.4. Since complement of an intuitionistic semi open set is intutionistic semi closed, A is ISC if and only if $I\,\mathrm{int}(Icl(A)) \subseteq A$.

Theorem 3.5. Let *A* be intutionistic semi open in the intuitionistic topological space (X,τ) and suppose $A \subset B \subset Icl(A)$, then *B* is intutionistic semi open.

Proof. There exist an intutionistic open set *U* such that $U \subseteq A \subseteq Icl(U)$. Since $U \subset B$, $Icl(A) \subset Icl(U)$ and thus $B \subseteq Icl(U)$. Hence $U \subset B \subset Icl(U)$ and *B* is intutionistic semi open.

Remark 3.6. If *U* is intuitionistic open in *X*, then *U* is intuitionistic semi open in *X*. Thus (X,τ) be an intuitionistic topological space. Let τ be the class of all intuitionistic open sets in *X*, then (i) $\tau \subset ISO(X)$ (ii) For $A \in ISO(X)$ and $A \subset B \subset Icl(A)$ then $B \in ISO(X)$. The converse need not be true.

Example 3.7. Let (X,τ) be an intuitionistic topological space. Let *X*={*a,b,c*} and τ is an intuitionistic open set containing

$$\tau = \left\{ \begin{matrix} \underset{\sim}{\varphi}, \underset{\sim}{X}, \langle X, \varphi, \{c\}\rangle, \langle X, \varphi, \{b,c\}\rangle, \\ \langle X, \{a,b\}, \{c\}\rangle, \langle X, \{a,b\}, \varphi\rangle \end{matrix} \right\} \text{then}$$

$A = \langle X, \{c\}, \{b\}\rangle$ is intuitionistic semi open but not an intuitionistic open set in *X*.

Theorem 3.8. A is intuitionistic semi closed if and only if there exists an intuitionistic closed set F such that $I\,\mathrm{int}\,F \subseteq A \subseteq F$.

Proof : Let A be intuitionistic semi closed, then $I\,\mathrm{int}(Icl(A)) \subseteq A$.

Let *Icl*(*A*) = *B*. Then *B* is intuitionistic closed. Now $I\ \mathrm{int}(B) \subseteq A \subseteq B.$

Thus if A is an intuitionistic semi closed set there exist an intuitionistic closed set F such that $I\,\mathrm{int}\,F \subseteq A \subseteq F$.

Conversely, let F be an intuitionistic closed set such that $I\,\mathrm{int}\,(F \subseteq A \subseteq F$.

Since $A \subseteq F \;\Rightarrow\; I\,cl\,(A) \subseteq I\,cl\,(F)$

$\Rightarrow\; I\,\mathrm{int}(I\,cl\,(A)) \subseteq I\,\mathrm{int}(I\,cl\,(F))$

= I int (F), because Fis intuitionistic closed , I cl(F) = F.

$\subseteq A$ [by hypothesis]

$\Rightarrow$ A is an intuitionistic semi closed set.

Remark 3.9 . An intuitionistic closed set is necessarily intuitionistic semi closed. The converse need not be true.

Example 3.10. Let (X,τ) be an intuitionistic topological space. Let *X*={*a,b,c*} and $\bar{\tau}$ is an intuitionistic closed set containing

$$\bar{\tau} = \left\{ \begin{matrix} \underset{\sim}{\varphi}, \underset{\sim}{X}, \langle X, \{b\}, \{a\}\rangle, \langle X, \{a\}, \{b\}\rangle, \\ \langle X, \{a,b\}, \{\varphi\}\rangle, \langle X, \{\varphi\}, \{a,b\}\rangle \end{matrix} \right\} \text{then}$$

$A = \langle X, \{\varphi\}, \{a\}\rangle$ is an intuionistic semi closed set but not an intuitionistic closed set in *X*.

Theorem 3.11. Let $A \subset Y \subset \underset{\sim}{X}$ where (X,τ) is an intuitionistic topological space and *Y* is intuitionistic subspace. Let $A \in ISO(X)$ then $A \in ISO(Y)$.

Proof. Let $U \subset A \subset Icl_X\,U$ where *U* is intuitionistic open in *X* and Icl_X denotes an intuitionistic closure operator in *X*. Now $U \subset Y$ and thus $U = U \cap Y \subset A \cap Y \subset Y \cap Icl_X U$ or $U \subset A \subset Icl_Y U$ Thus *U* is open in *Y* and the theorem is proved.

Remark 3.12. The converse of theorem 3.11 is need not be true.

Example 3.13. Let X be the intuitionistic space of X = {*a, b, c*} and $Y = \langle X, \{a\}, \{b\}\rangle$ and $A = \langle X, \{a\}, \{b\}\rangle$ then *A* is intuitionisitc open in *Y* and hence $A \in$ intuitionistic semi open (*Y*). But *A* is not intutionistic semi open in $\underset{\sim}{X}$.

Lemma 3.14. Let U be an intuitionistic open in *X*. Then *I cl* (*U*) – *U* is nowhere dense in *X*.

Proof. $I\ (U) = U \cup d(U)$ implies $Icl(U) - U \subseteq d(U)$ that is $Icl(U) - U$ consists of intuitionistic limit points of U. Let G be any non-empty intuitionistic open set in X. Then (1) $G \subseteq U$ implies $G \cap (Icl(U) - U) = \varphi$. (2) $G \cap U = \varphi$. Since G contains no intuitionistic points and also no intuitionistic limit points of U. Thus $G \cap (Icl(U) - U) = \varphi$. (3) If $G \cap U \neq \varphi, G \not\subset U$, Since G and U are intutionistic open sets, $G \cap U$ is a nonempty intuitionistic open subset of G and of U. Thus $Icl(U) - U \cap (G \cap U) = \varphi$. Therefore, for any nonempty intuitionistic open set of G, there exists a nonempty intuitionistic open subset of G which is disjoint from *Icl*(*U*)-*U*.

Remark 3.15. The converse of the above theorem is not true.

Example 3.16. Let us consider the ITS, (X,τ) where X = $\{a,b,c,d,e\}$ and

$$\tau = \left\{ \begin{array}{l} \underset{\sim}{\varphi}, \underset{\sim}{X}, \langle X,\{a,b,c\},\{e\}\rangle, \langle X,\{c,d\},\{e\}\rangle, \\ \langle X,\{c\},\{e\}\rangle, \\ \langle X,\{c\},\{d,e\}\rangle, \langle X,\{a,b,c,d\},\{e\}\rangle \end{array} \right\}$$

, if U = $\langle X,\{b,c\},\{d\}\rangle$ then I int (U) = $\langle X,\{c\},\{d,e\}\rangle$ and I cl (U) = $\underset{\sim}{X}$. Thereofore I cl(U) – U is nowhere dense in X.

Theorem 3.17. Let A be an intuitionistic semi open set in X, where X is an intuitionistic topological space. Then $A = U \cup B$ where
1. $U \in \tau$, 2. $U \cap B = \varphi$ and 3. B is nowhere dense.

Proof. $U \subset A \subset IclU$ for some intuitionistic open set U in X . But $A = U \cup (A - U)$. Let $B = A - U$ Then $B \subset IclU - U$ and thus nowhere dense by Lemma.3.14. Then A = U $\cup$ B, with $U \in \tau$ and $U \cap B = \varphi$.

Definition 3.18. An intuitionistic subset A of X containing x_0 is called an intuitionistic neighborhood of x_0 if A contains an intuitionistic open interval (a,b) containing x_0.

Definition 3.19. Let (X, τ) be an intuitionistic topological space and A be an intuitionistic subset of X. Then an intutionistic point $x \in X$ is called an intuitionistic limit point of A if every intuitionistic neighborhood of x contains an intutionistic point of A distinct from x.

Definition 3.20. The set of all intuitionistic limit points of a set S is called an intuitionistic derived set and is denoted by S' (or) $D(S)$.

Theorem 3.21. Let X be an intuitionistic topological space and $A = U \cup B$ where

1. $U \neq \varphi$ is intuitionistic open
2. A is intuitionistic connected and
3. $B' = \varphi$ where B' is the derived set of B, then $A \in ISO(X)$.

Proof . It is sufficient to show that $B \subset I\ cl\ (U)$. Suppose $B = C \cup D$ where $C \subset Icl(U)$ and $D \subset (Icl(U))^c$ and $D \neq \varphi$ [$(Icl(U))^c$ denotes the complement of Icl(U)]. Now $A = (U \cup C) \cup D$ and $U \cup C \neq \varphi$ by (1) and $D \neq \varphi$. Also $U \cup C \subset I\,cl(U)$ an intuitionistic closed set, and $D \subset B$, $D' \subset B' = \varphi$ and thus D is intuitionistic closed by (3) $D \cap Icl(U) = \varphi$.
Thus $U \cup C$ and D constitute a separation for A, this is a contradiction.

Remark 3.22. In general, the complement of an intuitionistic semi open sets need not be an intuitionistic semi open.

Example 3.23. Let X = {a, b, c},
$$\tau = \left\{ \begin{array}{l} \underset{\sim}{\varphi}, \underset{\sim}{X}, \langle X,\varphi,\{b,c\}\rangle, \langle X,\{a,b\},\{c\}\rangle, \\ \langle X,\{a,b\},\varphi\rangle \end{array} \right\}$$ then $A = \langle X,\varphi,\{c\}\rangle$ is intuitionistic semi open as well as complement of A also intuitionistic semi open. But the complement of $A = \langle X,\{a,b\},\{c\}\rangle$ is not.

Remark 3.24. In general, the intersection of an intuitionistic semi open sets need not be an intuitionistic semi open.

Example 3.25. Let $X = \{a, b, c\}$
$$\tau = \left\{ \begin{array}{l} \underset{\sim}{\varphi}, \underset{\sim}{X}, \langle X,\varphi,\varphi\rangle, \langle X,\{a\},\varphi\rangle, \langle X,\{b\},\varphi\rangle, \\ \langle X,\{c\},\varphi\rangle, \langle X,\{a,b\},\varphi\rangle, \langle X,\{a,c\},\varphi\rangle, \\ \langle X,\{b,c\},\varphi\rangle, \langle X,\{a,b,c\},\varphi\rangle \end{array} \right\}$$
then $A_1 = \langle X,\varphi,\{b\}\rangle$, $A_2 = \langle X,\varphi,\{c\}\rangle$ be two intuitionistic semi open sets but the intersection is not an intuitionistic semi open set.

Definition 3.26. The intuitinistic semi interior (I sint) of A is defined by the union of all intuitionistic semi open sets of (X,τ) contained in A.

Definition 3.27. The intuitinistic semi closure of A is defined by the intersection of all intuitionistic semi closed sets of (X,τ) containing A.

Lemma 3.28. Let τ be the class of intuitionistic open sets in the intuitionistic topological space X. Then $\tau \equiv I\ \mathrm{int}(ISO(X))$.

Proof . Let $U \in \tau$. Then $U \in ISO(X)$ so U = $I\ \mathrm{int}(U)$, $U \in I\,\mathrm{int}(ISO(X))$.
Conversely let $U \in I\,\mathrm{int}(ISO(X))$. Then $U = I$ int(A) for some $A \in$ ISO(X) and thus $U \in \tau$.

Theorem 3.29. Let τ and τ^* be two intuitionistic topologies for X. Suppose ISO (X, τ) $\subset$ ISO(X, τ^*). Then $\tau \subset \tau^*$.

Proof.

$\tau = I\,\text{int}(ISO(X,\tau)) \subseteq I\,\text{int}(ISO(X,\tau^*)) = \tau^*$

(by lemma 3.28)

Corollary 3.30. Let τ and τ^* be two intuitionistic topologies for *X*. Suppose ISO (*X*, τ) = ISO(*X*, τ^*). Then $\tau = \tau^*$.

IV. CONCLUSION

We conclude that the emergence of topology and its new results in the construction of some intuitionistic semi open concept will help to get rich results that yields a lot of hidden relations. The topological operators will play an essential role in data bases. In this paper, we give more specifically an overview of several results on intuitionistic semi open sets. The future application of this work will be useful in many fields.

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